Chapter 4

Tuning

> *"Have you noticed that when you've got a piano on its back and are installing a new set of casters, that someone will walk by and ask if you are tuning the piano? Then, when you are actually tuning the piano, they say, 'What are you doing?'"*
> —Barbara Richmond, RPT

Tuning the piano means adjusting the pitches of the strings so that all notes sound well when played, whether individually or together in various intervals and chords. To tune, one turns a tuning pin (or other tuning device) to alter the tension of the string and thus adjust the pitch.

In J.S. Bach's time (early 18th century), harpsichords, clavichords, and early pianos were tuned by the performer. Piano tuning became a profession in the 19th century, when piano's keyboard compass grew to over six octaves and triple stringing became prevalent. An increase in string tension caused corresponding increases in tuning pin torque and string friction, requiring a special technique to "set" the pin and the string. Equal temperament, which gained popularity during this period, required more precision, and therefore skill and experience, than had earlier temperaments.

Although piano tuning is clearly not something that can be mastered in an afternoon, anyone with normal hearing and coordination, a basic understanding of music theory, and adequate physical strength can learn to tune pianos with reasonable accuracy in a few months. Learning to tune is not unlike learning to play the piano—it requires *a lot* of practicing. If you aspire to tune on a professional level, be prepared for lengthy training.

This chapter, like the rest of this book, is written with the novice in mind. It contains detailed information about tuning aurally ("by ear") and with the help of a simple or advanced electronic tuning device (ETD). The chapter begins by explaining musical nomenclature, intervals, and equal temperament. Inharmonicity is discussed next, including its effects on coincident partials and interval stretch. Information about aural tuning is presented in the form of lessons, which should be practiced separately and systematically. The emphasis is on keeping things simple and getting results as quickly as possible, while providing enough information to develop your own system, and to explore more advanced techniques when you are ready. Tuning lessons are followed by instruction in electronic tuning and a discussion of special tuning techniques, such as raising and lowering pitch. The chapter closes with a list of issues that commonly affect piano tuning, and solutions to those problems.

To practice tuning, you will need a piano in a reasonably good condition. If the piano will be used between your practice sessions, you may need to hire a piano tuner to "undo" your work. Ironically, the type of piano you are most likely to have access to—a short grand or a vertical—is much harder to tune than a concert grand. Avoid pianos with very short strings, if possible.

I recommend learning from a piano tuner/technician. Even simply observing an experienced tuner at work will give you a clearer idea about recognizing beats, working with the tuning hammer, and setting the strings and tuning pins than will studying entirely on your own.

Be careful not to damage the piano, the floor, or the furniture, and avoid wearing clothes or apparel that could damage the piano's finish. If you need to remove pictures, vases, and other objects from the piano, return them in the same positions. Don't drag the bench across the floor. Lift it, or place a folded mover's blanket under it.

For instructions on removing case parts, see "Gaining Access" on page 136. If you need to open up the piano and do any work inside the action cavity, or on the baseboard in a vertical piano, look for signs of rodent infestation. If you find any, decontaminate and clean the piano before proceeding (see page 136).

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Table 3: Frequencies of Equal Temperament

Note	Octave								
	0	1	2	3	4	5	6	7	8
C/B♯	16.352	32.703	65.406	130.813	261.626	523.251	1,046.502	2,093.005	4,186.009
C♯/D♭	17.324	34.648	69.296	138.591	277.183	554.365	1,108.731	2,217.461	4,434.922
D	18.354	36.708	73.416	146.832	293.665	587.330	1,174.659	2,349.318	4,698.636
D♯/E♭	19.445	38.891	77.782	155.563	311.127	622.254	1,244.508	2,489.016	4,978.031
E/F♭	20.602	41.203	82.407	164.814	329.628	659.255	1,318.510	2,637.020	5,274.041
F/E♯	21.827	43.654	87.307	174.614	349.228	698.456	1,396.913	2,793.826	5,587.652
F♯/G♭	23.125	46.249	92.499	184.997	369.994	739.989	1,479.978	2,959.955	5,919.911
G	24.500	48.999	97.999	195.998	391.995	783.991	1,567.982	3,135.963	6,271.927
G♯/A♭	25.957	51.913	103.826	207.652	415.305	830.609	1,661.219	3,322.438	6,644.875
A	27.500	55.000	110.000	220.000	440.000	880.000	1,760.000	3,520.000	7,040.000
A♯/B♭	29.135	58.270	116.541	233.082	466.164	932.328	1,864.655	3,729.310	7,458.620
B/C♭	30.868	61.735	123.471	246.942	493.883	987.767	1,975.533	3,951.066	7,902.132

tuning lessons below. See page 100 for information on how to calculate the frequencies and beat rates within the equal temperament. For your convenience, Table 3 lists the theoretical frequencies of all notes on the piano. You will learn to tune equal temperament in “Lesson 7” on page 117.

Inharmonicity

The difficulties imposed by equal temperament are further aggravated by *inharmonicity*, which raises the pitch of overtones. Instead of being true multiples of the string’s primary frequency, as they theoretically should be, overtones get progressively higher (Figure 210). This has far-reaching consequences.

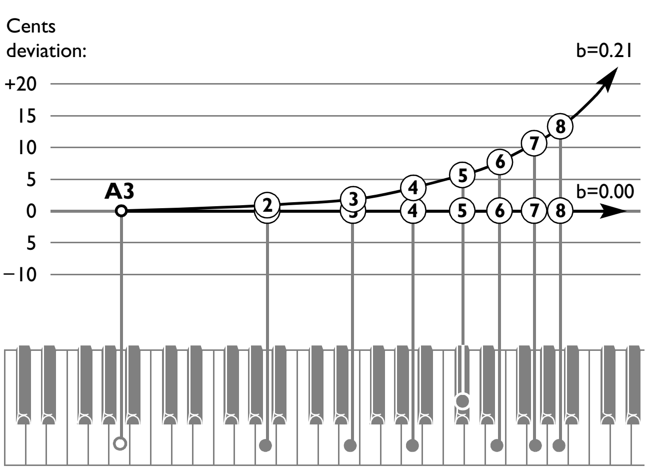

Figure 210 *Inharmonicity raises the frequency of partials exponentially. As the curved line illustrates, the eighth partial of note A3 in a 1923 6′4″ [193 cm] Steinway A is over 13 cents higher than it would be without inharmonicity. It is not unusual for the inharmonicity of that note to be twice as high in some pianos. (The visualization is based on inharmonicity charts in Daniel Levitan,* The Craft of Piano Tuning.*)*

The classical explanation for inharmonicity is that the vibrating string breaks up into partial-generating segments that behave as virtual strings—shortened, but just as thick as the whole string. The segments are increasingly stiffer, and the stiffer they are, the more their ends resist bending. This, in turn, reduces each segment’s effective length, and that causes the pitch to rise.[197]

As Dave Carpenter, RPT, points out, a more accurate explanation is that vibrations move through the string as periodic (repeating) waves of energy, at a certain speed or *wave velocity*. Each partial has its own *wavelength*, which is in inverse relationship with its frequency—the higher the partial, the shorter the wavelength. In an ideal string with no stiffness, wave velocity would be constant for all partials, and they would coincide with the harmonics (mathematically perfect partials). However, an actual piano string is stiff to a certain degree (the thicker it is, the stiffer it will be), and stiffness makes it resist bending. This, in turn, creates *bending force*, which increases the speed with which the wave moves through the string. To understand bending force, imagine bending a short and a long wire of the same thickness. The short wire will resist the bending more and spring back with greater force and speed than the longer wire. Bending force increases as wavelength decreases, which means that wave velocity increases toward high partials. In effect, the wire exhibits greater stiffness at shorter wavelengths. This is why the frequency shift is progressive—the higher the partial, the more inharmonic it is. See the sidebar “Calculating Pitch Raise of Partials” on page 106 for sample data.

Whereas the *prominence* of partials defines tone quality or timbre, inharmonicity imparts its own “flavor,” which can make the piano sound noisy and jarring (greater inharmonicity) or focused and clear (lower inharmonicity). High inharmonicity can make low bass notes sound downright gong-like.

The tonal effect of inharmonicity changes during the life of the note because high partials die out more quickly

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[197] See W.V. McFerrin, *The Piano: Its Acoustics*, pp. 38–40.

Table 4: Inharmonicity and How Much It Raises Partials[a]
(1923 6′4″ [193 cm] Steinway A III)

Note	Freq. (Hz)	*b*	Partial 2.		Partial 3.		Partial 4.		Partial 5.		Partial 6.		Partial 7.		Partial 8.	
			cents	bps	cents	bps	cents	bps	cents	bps	cents	bps	cents	bps	cents	bps
A0	27.5	0.370	**1.50**	0.05	**3.50**	0.17	**6.50**	0.41	**12.50**	1.00	**14.0**	1.34	**19.0**	2.12	**23.5**	3.01
A1	55	0.160	**1.00**	0.06	**3.60**	0.34	**3.70**	0.47	**5.60**	0.89	**5.60**	1.07	**7.60**	1.69	**8.20**	2.09
A2	110	0.055	**0.20**	0.03	**0.40**	0.08	**0.30**	0.08	**0.80**	0.25	**1.90**	0.73	**2.90**	1.29	**3.70**	1.88
A3	220	0.210	**0.84**	0.21	**1.89**	0.72	**3.36**	1.71	**5.25**	3.34	**7.56**	5.77	**10.29**	9.18	**13.43**	13.71
A4	440	0.707	**2.83**	1.44	**6.36**	4.86	**11.32**	11.54	**17.68**	22.58	**25.46**	39.11	**34.65**	62.27	**45.26**	93.24
A5	880	2.294	**9.18**	9.36	**20.65**	31.68	**36.71**	75.44	**57.36**	148.22	**82.60**	258.02				
A6	1760	6.484	**25.94**	53.14	**58.36**	181.02	**103.75**	434.79								
A7	3520	21.561	**86.25**	359.62	**194.05**	1252.53	**344.99**	3,104.87								

[a] Bass string values (notes A0–A2) are averages of multiple readings with Verituner® 4.2.3 on an Apple® iPhone® 4S (the lowest partials, which couldn't be measured, were estimated; an average coefficient of inharmonicity, *b*, was estimated from the measurements); steel string (A3–A7) values were calculated using the formula in Robert Young, "Inharmonicity of Plain Wire Piano Strings," Equation 9. The calculations were confirmed to be very close to measurements with Verituner. For supporting data and calculations, see "Inharmonicity Measurements" at `http://www.pianosinsideout.com/bonus`.

than low ones. This gives the piano tone a dynamic quality in which the decrease in loudness is accompanied not only by a decrease in brightness but also a slight change in perceived pitch. The effect is especially noticeable in low bass strings, which, as Daniel Levitan, RPT, points out, have "a curious quality of continuously going flat without ever changing pitch."[198]

Inharmonicity is lower overall in long pianos than in short ones. It tends to be low in the middle section, and to increase rapidly toward the top note. It also increases, though typically to a much smaller degree, from the highest to the lowest wrapped strings. Pianos that have a significant break in inharmonicity between wrapped and plain strings in the low tenor (the highest wrapped strings having lower inharmonicity than the lowest plain strings) are hard to tune in that section.

Inharmonicity is not fully predictable in the bass section because bass strings are made by wrapping a copper wire around a steel core (in many pianos, the lowest strings are wrapped with two copper windings), and the wrap is subject to variations in length, tightness, and elongation of the copper wire. Other factors that make inharmonicity unpredictable are length of unwrapped ends, length of single-wrapped segments in double-wrapped strings, and length and extent of swaging—as well as factors outside of the string, such as stiffness and mobility of string terminations, and soundboard and case resonances.

Effects on Tuning

Figure 211 illustrates a typical progression of inharmonicity in a well-scaled, medium-size grand piano—it increases from fairly low values at the lowest plain steel strings in the tenor (B♭2 in this piano) to very high values in the treble ("*b*" values indicate the coefficient of inharmonicity, which is used to calculate the pitch shift of all partials—see the sidebar "Calculating Pitch Raise of Partials" on page 106). This causes a corresponding increase in stretch toward the top note.

Although inharmonicity shifts partials by a large amount in the treble, this is not as much of a problem as it may seem. As you can see in Figure 213, treble notes generate few partials, even when played loudly. Those partials decay rapidly, and only the first and second partials remain audible. As a result, the treble octaves can be tuned on the 2:1 level. When those pairs of partials are tuned beatless there are no other coincident pairs to generate beats, and the interval sounds in tune.

It is much more important that the prominent coincident partials of *all* octaves form a smooth, uninterrupted curve. When they do, tuning is easy because all audible coincident partial pairs are nearly beatless. Compare, for example, the prominent partials of the notes A4 and A5, indicated by the dots on the A4 and A5 curves in Figure 211. The curves are difficult to discern because they practically cover each other. As you can see, the two notes are tuned beatless on the 4:2 level, but the 2:1 and 6:3 levels are also almost beatless. A scale design with such closely matched inharmonicity curves permits the tuning of octaves of exceptional purity. The other intervals, such as P5s, P4s, M3s, etc., are also easy to tune because their beat rates are clearly discernible. In our sample piano, the curves "fit" almost perfectly in all notes from A2 to A7.

Note the curve in Figure 212, which connects the first partials of the notes charted in Figure 211.[199] If it were practical for the strings in the bass section to continue increasing in length and thickness at the rate at which they do from the treble to the tenor, this curve would continue flattening out toward the lowest string, which would have

[198] Daniel Levitan, *The Craft of Piano Tuning*, p. 46.

[199] This curve looks just like the curve that O.L. Railsback presented to the American Acoustical Society in 1938, plotting average frequencies of first partials measured in multiple pianos with a chromatic stroboscope. See W.V. McFerrin, *The Piano: Its Acoustics*, pp. 41–42.

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Calculating Pitch Raise of Partials

If you know a string's coefficient of inharmonicity (*b*), which can be measured with some electronic tuners or calculated using Robert Young's formula (available at `http://www.pianosinsideout.com/bonus`), you can calculate the amount of **pitch raise for each partial**, in cents, with this formula:

25 $$r(\text{cents}) = b \times \text{partial number}^2$$

For example, knowing that in the above piano the note A3 has a *b* of 0.21, we can quickly find out that its third partial is raised by 1.89 cents:

26 $$r = 0.21 \times 3^2 = 0.21 \times 9 = 1.89 \text{ cents}$$

To translate that to beats per second, we first calculate the **theoretical frequency** of the third partial with this formula:

27 $$f(\text{Hz}) = f_{\text{first partial}} \times \text{partial number}$$

Without inharmonicity, the frequency of the third partial would be 660 Hz:

28 $$f = 220 \times 3 = 660 \text{ Hz}$$

Next, we use Equation 15 on page 100 to calculate the **frequency increase** of the third partial from the number of cents (1.89) we calculated in Equation 26, and we get 660.721 Hz:

29 $$f = 660 \text{ Hz} \times 2^{\frac{1.89}{1200}} = 660 \text{ Hz} \times 1.00109 = 660.721 \text{ Hz}$$

And finally, we calculate the **beat rate** between the inharmonicity-raised and theoretical frequencies of the third partial of A3 using Equation 21 on page 100:

30 $$n = 660.721 - 660.000 = 0.721 \text{ bps}$$

See Table 4 for pitch-raise values for up to eight prominent partials of each note A in the above piano. These values are plotted in Figure 211 on page 104. To calculate the frequency of each partial, add bps to the theoretical frequency calculated with the formula in Equation 27.

In the octave between A2 (b=0.055) and A3 (b=0.210) the multiplier is 3.82, very close to 4. There is practically **no interval inharmonicity** (see Figure 214B), and almost no stretch between A2 and A3 (Figures 211 and 212).

But what if the multiplier was 5, 6, or more? In our sample piano, A2 is the highest note strung with wrapped strings on the tenor bridge,[203] but the speaking length of those strings is almost identical to the speaking length of A♯2. Because the wrapped strings of A2 have a thinner steel core than the unwrapped strings of A♯2, their inharmonicity coefficient, *b*, is significantly lower than the coefficient of the A♯2: 0.055 vs. 0.155. This sudden drop in inharmonicity across the break between steel and wrapped strings causes **negative interval inharmonicity** and a negative stretch, which means that instead of being wid-

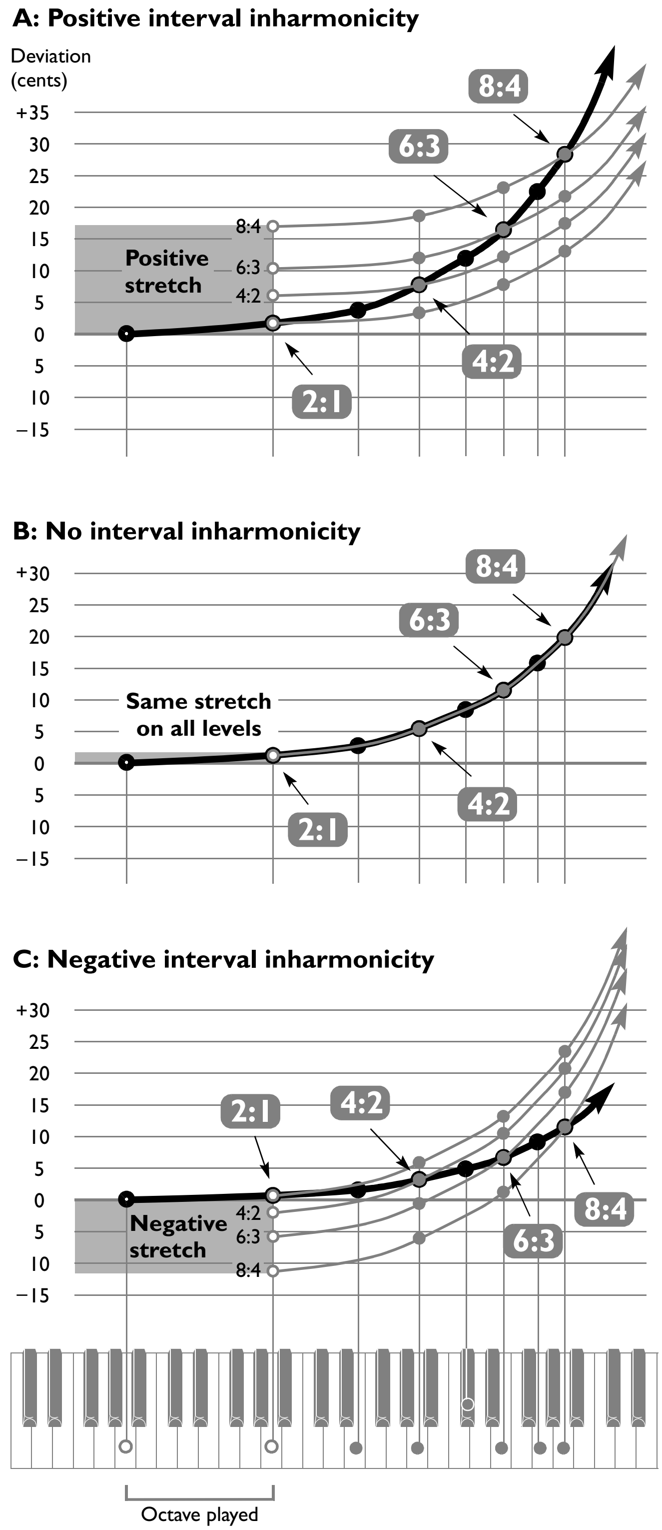

Figure 214 Interval inharmonicity and octave stretch. The black curve indicates the lower note, and the gray curves show four possible tunings of the upper note. ***A****: coefficient of inharmonicity* b *of upper note is less than 4 × b of the lower note.* ***B****: upper note b = 4 × lower note b.* ***C****: upper note b > 4 × lower note b. Note that in case C the stretch is positive at the 2:1 level, but negative for the 4:2, 6:3, and 8:4 levels.*

[203] The five lowest unisons in the tenor section are strung with wrapped bichords.

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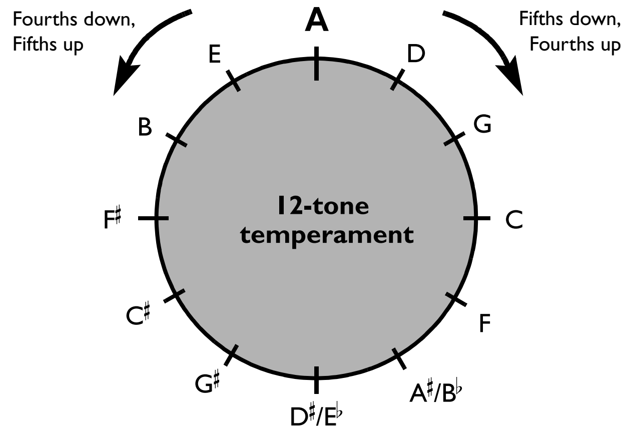


Figure 233 *The 12-tone temperament circle (circle of fifths).*

beating intervals as octave tests; Lesson 9 shows you how to use them to improve your temperament tuning.

To tune this sequence successfully, you need to *tune quickly.* You will need to tune six intervals in one direction and five in the other to learn whether your beat rates are too slow or too fast. That is a lot of intervals. Your tuning technique is likely to add instability, and you may need to repeat this sequence several times. Limit the amount of time you spend on each note, even if you know you are accumulating errors. Establish a rhythm to outpace fatigue and frustration. Adjust the pitch, set and pound, and test. Repeat if needed and move on.

Sit at the piano comfortably and don't move your head while listening to beat rates. Keep in mind that you are learning several skills simultaneously—listening to intervals is just one of them. If you get frustrated, step away, and return to the piano after some rest.

Press and hold the damper pedal, and **insert the strip mute** throughout the middle section (approximately E3 to C5, see Figure 227 on page 113). Check the A4 against the tuning fork or an electronic tuner. If it beats under 2 bps, tune it beatless to the fork; otherwise, either settle for the existing pitch, or first pull up/lower the pitch of the entire piano with an ETD (page 128). Changing the pitch by more than a few cents will destabilize the tuning and undermine your efforts.

The tuning sequence is listed step by step in "Simple Temperament Tuning Sequence Using Slow-beating Intervals." The sequence starts by tuning the middle A to the fork (see Lesson 2 above), and establishing a P5 and P4 in the octave below (steps 1–3). The octave should be just *slightly* wide but without perceptible beats, and the P4 should beat just slightly faster than the P5, both under 1 bps. You can spend a little more time on these two notes because they will serve as aural templates for the beat rates of the other P5s and P4s you will tune. Tune the next five notes to D♯4 by tuning P4s up P5s down (steps 5–9). Make the beat rates the same as the P5 and P4 you tuned initially. Next, start at the middle A again and tune the five notes in the opposite direction of the temperament circle (steps 10–15). Compare the last note, G♯4 with D♯4 (step 15). If this P4 is wider than beatless but beats under 1 bps, congratulations! If it beats at more than 1 bps, it is too wide—increase the beat rate of the P4s and P5s a touch in both directions. This will narrow the P5s and widen the P4s, and raise the notes tuned during the first half cycle, including the D♯4, while lowering the notes in the second half cycle. If the P4 D♯4-G♯4 beats at less than 1 beat in 2 seconds, doesn't beat at all, or is narrower than beatless, decrease the beat rates of the P4s and P5s. Make corrections until all the P4s beat at approximately the same rate, slightly faster than the P5s.

Lesson 8: Complete Tuning Procedure—Aural Tuning

First, tune the temperament as explained in Lesson 7 or Lesson 9. Leave the strip mute in place and tune the octaves up chromatically from the temperament to the top note, then down to the lowest note. More detailed instructions and test intervals for each section are given below.

When tuning the octaves, tune *all* the strings in a unison before moving to the next note. Use a rubber mute or a Papps mute to mute the unwanted string(s). Use *test intervals* to improve the precision of your octave tuning (page 120).

When you finish, check the temperament and correct it if necessary, then correct all the corresponding octaves. Remove the strip mute and tune the unisons in the temperament section. Recheck the tuning by playing the intervals you used to tune the temperament, and make corrections as necessary. Use rubber or felt mutes to isolate individual strings and "repair" the unisons, always double-checking the temperament.

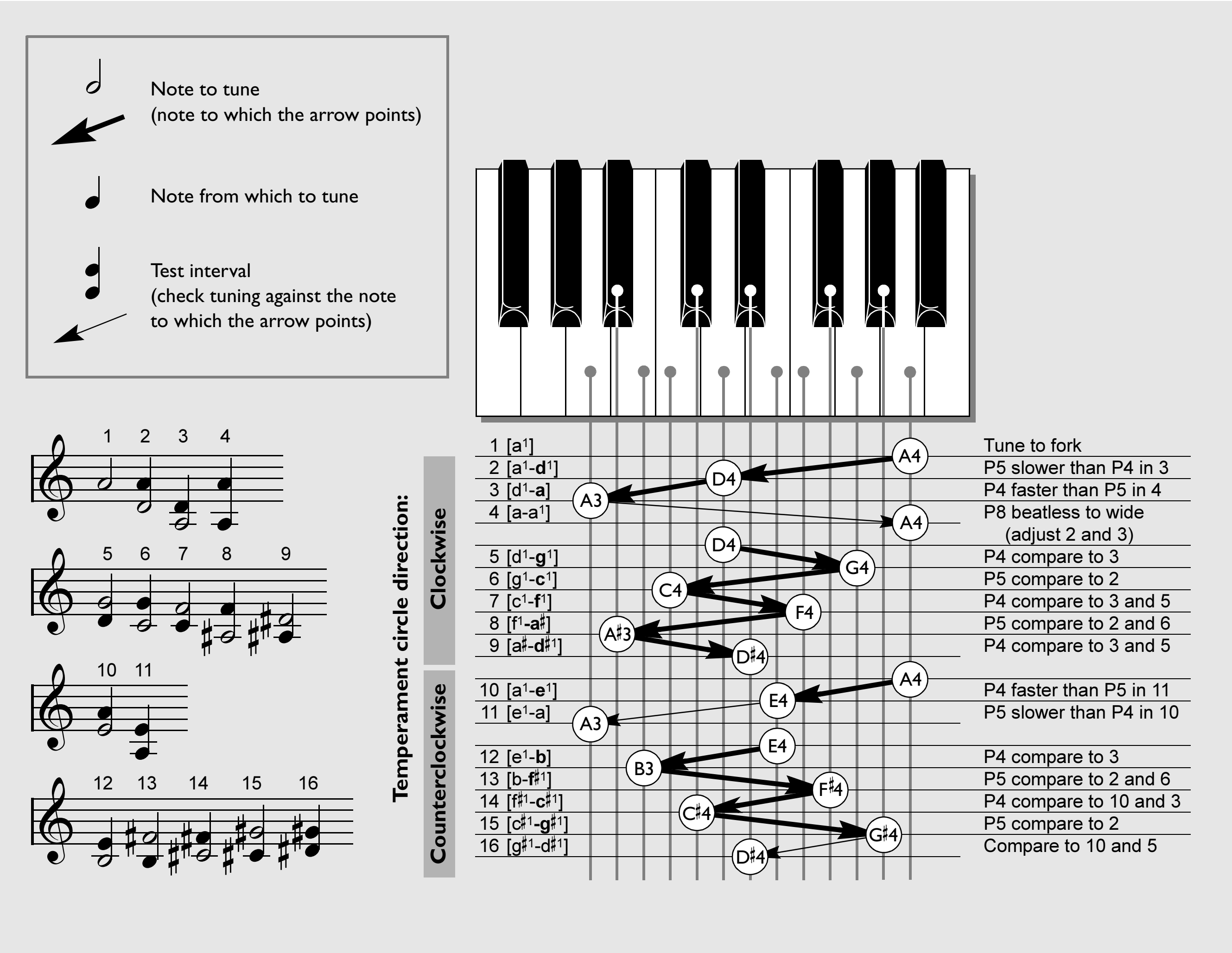
Which String to Tune First in Unisons?

When you tune whole trichord unisons, tune the shortest string first: the left string in grands, the right string in verticals. The shorter the string between the tuning pin and its speaking length, the more readily it responds to tuning hammer movements.[217] The short string also isn't affected when the middle and longest strings bend around it (see Figure 244 on page 132). Tune the middle string to the short string next, then tune the remaining string to the first two. For the notes next to plate struts, it is more practical to start with the string that is closest to the strut, regardless of its length.

You can tune the bichords (all of which are typically wound strings) from right (longer) to left (shorter), in both grands and verticals. String lengths matter less in bichords, whereas reducing how much you handle the mute saves time.

[217] In verticals it may be affected more than the left string by temperature swings during tuning.

Simple Temperament Tuning Sequence Using Slow-beating Intervals



Stretching the Octaves

When you tune octaves in the middle section to sound pure and beatless, double and triple octaves will sound narrow in all but the longest pianos. Address this by slightly stretching the single octaves, but without making them beat perceptibly. Use octave-stretch test intervals to check the octave width (page 120). The stretch is more a sense of widening an otherwise pure octave than detuning it. With somewhat wide octaves, a piano's sound is richer, more resonant, more "open."

Of course, not only are multiple octaves affected by insufficient stretch, but the beat rates in all narrow intervals (P5s, m3s, and m6s) become objectionable. On the other hand, overstretching the octaves will make the wide intervals (P4s, M3s, and M6s) too noisy.

The amount of stretch increases toward the high treble not only because inharmonicity increases most in that section, but because we perceive high pitches as lower than they really are. Some tuners tune the highest notes so sharp that the last few octaves sound like m9s. Most pianists don't find overstretching the top to be satisfying. To blend those notes with the rest of the piano, test them using multiple octaves and other compound intervals (for example, P5s and P4s over two octaves).

Tuning the Middle and Tenor Sections

At first, use **P4**s and **P5**s as your main test intervals. When a P4 is above a P5 within an octave, it should beat slightly faster than the P5. When it is below a P5, they should have about the same beat rate.

Start using fast-beating test intervals, such as **contiguous M3**s, **adjacent M3**s, and octave tests, such as the **M3/M10**. Lesson 9 introduces some of these and "Test Intervals" on page 120 explains how they work.

Tuning the Bass

Tune the bass section using descending **octaves** as the primary interval. Listen for the *underlying* beats, heard as relatively long waves, and ignore the confusing rapid beats among the higher partials. Whenever in doubt, drop the pitch of the lower note in an octave until you can clearly hear the low beats ("woo woo"), pull it up until the beats

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